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LETTER TO THE EDITOR

The crumpled state of some non-equilibrium fractal surfaces

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Abstract. We study how the three-dimensional 'air' or Pythagorean distance $r(Q, Q')$ between two points Q and Q' on a non-equilibrium crumpled fractal surface (CS), with the topology of the plane, transforms in the internal or geodesic distance $x(Q, Q')$—with probability $P(x, r)$—after the unfolding of the CS on a plane. The probability distribution $P(x, r)$ governing this process is examined for the first time. Among other results we find that (1) the width of $P(x, r)$ 'diverges' for r near the ensemble average radius R of the CS and (2) $\langle x\rangle \sim r^{1/3}$.

The last few years has witnessed a renaissance in the study of random systems, mainly in connection with fractals and non-equilibrium phenomena [1]. In particular, very simple experiments [2] and computer simulations [3] have revealed the existence of several types of non-equilibrium critical phenomena associated with many kinds of random fractals. On the other hand, one of the major challenges in theoretical physics today is to understand the properties and behaviours of surfaces and membranes. From the experimental point of view, much evidence has been reported about the possible relevance of a fractal description for surfaces of materials involved in many physical, chemical and biological processes, e.g. adsorption [4], fractures [5], catalysis [6] and protein and antibody specificity and recognition in biomolecular interactions [7].

In this letter we deal with non-equilibrium configurations of fractal crumpled surfaces (CS) obtained from random and irreversible compactification of two-dimensional manifolds. Some properties of these random objects have been studied recently [8]. The CS are self-avoiding surfaces with the topology of the plane and with spherical shape. They satisfy the mass-size scaling relation mass $\sim L^2 \sim R^D$, where L is the linear (uncrumpled) size of the manifold, R is the ensemble average radius or the radius of gyration of the CS and $D = 2.5$ with typical fluctuations of 8% [8]. The existence of a narrow interval of values for the exponent D in these ill-defined crumpling operations is an empirical fact probably correlated to topological constraints, i.e. surface connectivity certainly implies quite limited crumpling procedures which in turn lead to an almost invariant value for D. Since D for CS is unaffected by the way of crumpling (with pressure applied, in haste or not) [8], it is possible to reproduce these objects and to study their geometrical and physical properties [8, 10, 11, 14].

The formation of irregular fractal patterns under non-equilibrium conditions has become a subject of great scientific and practical interest [1-3, 8-11]. The CS studied here belong to an ensemble completely different from the equilibrium ensembles of random surfaces recently examined by a number of authors [12]. The statistics of random surfaces and of the CS in particular are still to a large extent an unexplored field. Besides representing an interesting problem in surface statistics on their own, CS

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can also be of potential interest in polymer and surface physics. From the conceptual point of view CS may display novel critical phenomena which are at this point poorly understood [8, 13]. In this letter we discuss a new kind of critical phenomenon involving this class of surfaces. It is expected that many problems on diffusion [10], capillary and screening effects [14], adsorption and chemical kinetics [6], vibrations and mechanical properties in general [11] can be studied in connection with CS.

As a consequence of the random compactification processes which generate the CS, the 'air' distance $r = r(Q, Q')$ between two points Q and Q' belonging to the manifold in the tridimensional space (i.e. in the crumpled C-state) is transformed after the unfolding of the CS on the plane (flat F-state) in some distance $x = x(Q, Q')$ with a probability $P(x, r)$ dependent on x and r. Here the power laws associated with this type of critical phenomenon are investigated and the probability distribution P is shown for the first time. Among other results, we find that: (i) the width of the probability distribution P 'diverges' when r approximates a critical value close to R; (ii) the ensemble average $\langle x \rangle$ for the distances in the F-state scales with r as $\langle x \rangle \sim r^{1/3}$.

Although a lot of theoretical results have been published on random surfaces at equilibrium [12], the same cannot be said about the non-equilibrium crumpled surfaces. To gain more insight and knowledge, the probability distribution P introduced in the previous paragraph is studied in this letter from the experimental point of view using an ensemble of 300 square sheets of paper with edge $L = 66.0$ cm. After the crumpling of each one of these sheets into approximately spherical balls, the CS presented an ensemble average radius $R = 4.3 \pm 0.2$ cm. On the external surface of each CS of the ensemble we marked at random 10 pairs of points with the distances r between them satisfying $100\, r/R = 4.63$, 6.95, 9.27, 13.9, 18.5, 27.8, 34.8, 57.9, 92.7 and 185. Thus r varies by a factor of 40 which represents the largest internal of variability for this length in this type of experiment. In the second stage the CS were unfolded on the plane (without tears) and the distribution of distances x as a function of the fixed values of r was examined. An internal distance between two points Q and Q' at the F-state is computed as x if both points are on the same face of the manifold or on opposite faces of the manifold.

In figure 1 the average distance $\langle x \rangle$ (at the F-state) corresponding to a fixed distance r (in the C-state) is plotted against r. It is found that $\langle x \rangle \sim r^{0.333}$. An estimate for this exponent may be obtained with the following argument. (i) The intersection of a CS of fractal dimension $D \simeq 2.5$ with a straight line segment of length r (the Pythagorean

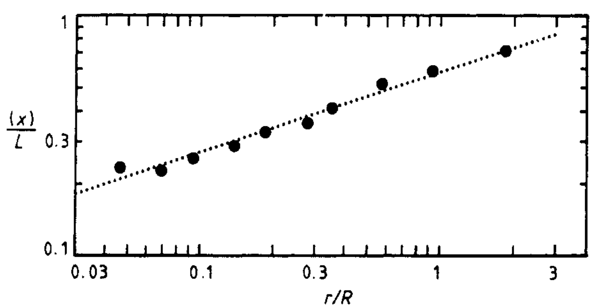

Figure 1. The (ensemble average) 'internal' 2D distance $\langle x \rangle$ between two points, Q and Q', belonging to a CS as a function of the 'air' 3D distance $r = r(Q, Q')$. $L = 66$ cm is the size of the manifold and $R = 4.3$ cm is the radius of gyration. $\langle x \rangle \sim r^{0.333}$ with a coefficient of correlation 99%.

distance) is a Cantor set S of dimension $\delta = D-2 \simeq 0.5$. (ii) Uncertainty in $\delta =$ uncertainty in $D = \pm 0.20$ or $\pm 8\%$ of D [8]. (iii) The expected (effective) mass(m)-size(r) scaling relation for S is $m \sim r^{0.5\pm0.2}$. m can be estimated in a mean-field manner by smearing out all the geodesic length $\langle x\rangle$ over the length r, i.e. $m \sim \langle x\rangle \sim r^{0.5\pm0.2}$.

The relative fluctuations of the distance x at the F state, $x(\text{RMS})/\langle x\rangle$, scales as $x(\text{RMS})/\langle x\rangle \sim r^{-0.344}$. The root mean square distance $x(\text{RMS})$ is defined by

$$x(\text{RMS}) = \left\{ \sum_{i=1}^{N} (x_i - \langle x\rangle)^2/(N-1) \right\}^{1/2}$$

with $N = 300$ and x_i being the Euclidean distance QQ' at the F-state for the ith CS, for a fixed value of r. Thus, the relative fluctuations in x decrease with a one-third power law as r increases.

The probability distribution P as a function of x is shown in figure 2(a–d) for some chosen values of r. In these figures we have expressed x as a new discrete variable z. Thus $z = n =$ *integer* means that $(n-1)\varepsilon \leq x/L < n\varepsilon$, for $1 \leq n \leq \sqrt{2}/\varepsilon$, $L = 66$ cm. A statistical analysis of the data was made with the values of $\varepsilon = 0.014\,14$, $0.030\,09$, $0.058\,93$, $0.101\,02$, $0.176\,78$, $0.262\,84$ and $0.471\,40$ in conformity with the usual procedure to choose the bin sizes [15]. Figure 2 corresponds to $\varepsilon = 0.058\,93$, i.e. the largest interval of variation of x/L, $0 \leq x/L \leq \sqrt{2}$, is covered by $n = \sqrt{2}/\varepsilon = 24$ subintervals (bins) of identical size. The width w at half-height of the probability distribution (indicated by triangles in figure 2) assumes their largest value for $r/R \simeq 1$. This happens for all the values of ε in our experiments. The dependence of w with r, is shown in figure 3 for some values of the bin size ε. Thus: (i) w is nearly constant for $0.04 \leq r/R \leq 0.4$,

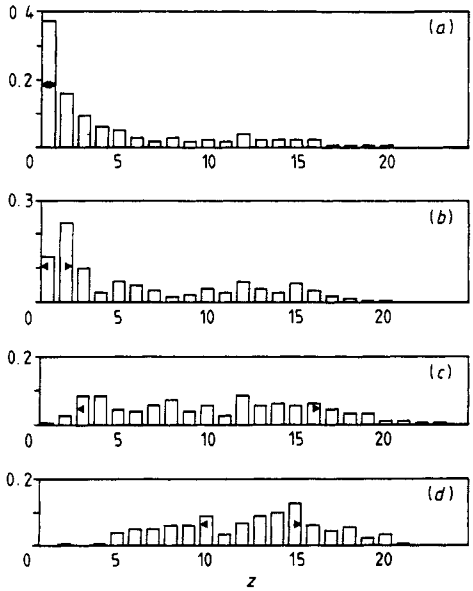

Figure 2. The probability distribution $P(x, r)$ as a function of the dimensionless variable x/L, $L = 66$ cm ($x/L =$ integer z means that $(z-1)\varepsilon \leq x/L < z\varepsilon$, where ε is the bin size; $\varepsilon = 0.058\,93$ in these figures, thus, the interval $0 \leq x/L \leq \sqrt{2}$ is covered by 24 of such bins) for $r/R = 0.069\,51$ (a), $0.278\,04$ (b), 0.927 (c) and 1.85 (d), $R = 4.3$ cm. Triangles denote the width of P at half-height. See text and figure 3.

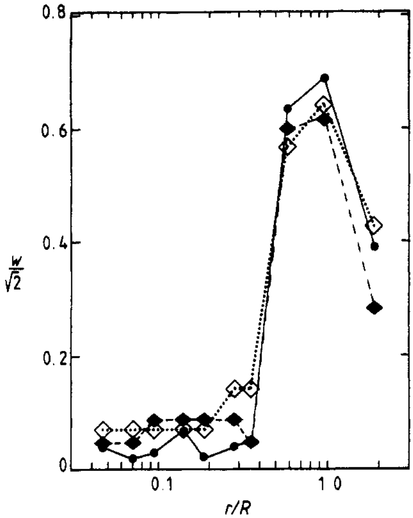

Figure 3. The width (x) of the probability distribution P at half-height as a function of the same dimensionless variable r/R of figures 1 and 2 for $\varepsilon = 0.014\,14$(●), 0.058 93(◆) and 0.101 02(◇). w increases rapidly for $r/R \simeq 0.4$ and reaches the maximum (about 65% of the largest possible value for w) for $r/R \simeq 1$, irrespective of ε.

$R = 4.3$ cm; (ii) w increases by a factor of nine in the interval $0.4 \leqslant r/R \leqslant 1$; (iii) w decreases rapidly for $r/R > 1$, independent of ε.

Furthermore, we have examined how $P_{\max}(r)$, the maximum in $P(x, r)$ depends on r, and ε. $P_{\max}$ exhibits the power-law scaling $P_{\max} \sim r^{-\alpha}$, with $\alpha = 0.51 \pm 0.04$ irrespective of ε. On the other hand, $P_{\max} \sim \varepsilon^{0.61 \pm 0.11}$, irrespective of r in the interval $0.046 \leqslant (r/R) \leqslant 1.85$.

It is interesting to note that CS, percolation clusters and DLA [1] are examples of random fractals which look completely different from each other but have $D \simeq 2.5$ when $d = 3$. Thus, it is important to investigate the fractal dimension $d_{\min}$ of the minimum path on the CS [1, 16]. In this case the Pythagorean distance r scales with the minimum path $l = x_{\mathrm{minimum}}$ as $l \sim r^{d_{\min}}$. For CS we find that $d_{\min} = 1$ within fluctuations of 10%. This value in $d = 3$ is comparable to thhose obtained for DLA and percolation clusters, namely $d_{\min} = 1$ (independent of d) and $d_{\min} = 1.3 (d = 3)$, respectively [1].

The statistics of random surfaces, and of the CS in particular, are still to a large extent an unexplored field. The experiment described in this letter explores the special features of the probability distribution which correlates the three-dimensional external 'air' distance $r(Q, Q')$ with the uncrumpled (geodesic) distances $x(Q, Q')$ between two points Q and Q' belonging to non-equilibrium crumpled (fractal) surfaces with the topology of the plane. A complete analysis of the problem discussed in this letter seems difficult from the point of view of theory and computer simulations. However, we expect that the scaling relations describing this interesting critical behaviour will stimulate theoretical work as well as numerical simulation on the subject. Furthermore, we think that the ideas developed here are also useful in the study of other issues on crumpled surfaces and random geometries.

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